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DELAY EQUATIONS WITH A CONTROL PARAMETER ARISING IN FLOTATION PROCESSES

Daniela D. Parashkevova
Department of Automation of Mining Production, Faculty of Mine Electromechanics,
University of Mining and Geology "St. I. Rilski", Sofia, BULGARIA

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ABSTRACT
The paper deals with a general control problem modelling problems arising in some mineral and flotation processes. Mathematical formulation of the problem leads to differential system with delay and a control parameter. We reduce the problem to an operator equation in a suitable function space and by means of fixed point theorem find a solution. The solution obtained can be approximated by a sequence of successive approximations.

KEYWORDS: Flotation Process, Control Parameter, Delay Differential Equations, Fixed Points in Uniform Spaces.

1. INTRODUCTION
The main purpose of the present paper is to propose a general mathematical formulation of problems arising in the control of some mineral and flotation processes of mining production, considered in (Prozuto, 1987; Cipriano; Wills et al., 2016; Speedy et al., 1970; Parashkevova, 2006). In many cases, the really existing delays in feedback processes cannot be ignored. That is why we describe the dynamics of these processes using differential equations with delays. We formulate the main problem: to find a solution of the vector delay differential equation continuously depending on a control parameter u and to find a such particular value of u that this solution to attend prescribed value x_T at a prescribed instant *T*:

$$\dot{x}(t) = F(t, x(t-\Delta_1(t)),..., x(t-\Delta_m(t)), u), \quad t \in [0, \infty), \tag{1}$$

$$x(t) = \varphi(t), \quad t \in (-\infty, 0], \tag{2}$$

$$x(T) = x_T \ . \tag{3}$$

Consider the n-dimensional Euclidian space R^n with some norm $\|.\|$ and the bounded subset $U \subset R^n$.

Here $x(.) \in R^n$ and $F(s, p_1,..., p_m, u) : [0, \infty) \times R^n \times ... \times R^n \times U \to R^n$ is a continuous function satisfying conditions formulated below; $\varphi(t) : (-\infty, 0] \to R^n$ is a prescribe initial function depending on the control parameter; $-T_0 = \min\{t-\Delta_1(t),..., t-\Delta_m(t) : t \in [0, \infty)\}$, where $\Delta_k(t) : [0, \infty) \to (-\infty, \infty)(k = 1,2,..., m)$ is a delay function and $x_T \in R^n$ is a prescribed vector.

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2. EXISTENCE-UNIQUENESS OF SOLUTION OF THE INITIAL VALUE PROBLEM DEPENDING ON PARAMETER

Here we reduce the above problem to an operator form following (Angelov, 2009; Mossiagin, 1970; Angelov, 1987; Angelov, 1981; Angelov, 2014). We present the initial value problem (1), (2) in the following operator form

$$x(t)=\int_0^t F(s,x(s-\Delta_1(s)),...,x(s-\Delta_m(s)),u)ds,$$

$$x(t)=\varphi_0(t), t\le 0$$

where $\varphi_0(t)$ is a prescribed initial function, assuming $\varphi_0(0)=0$.

Let the following conditions (BU) be fulfilled:

1) The continuous function $F(s,p_1,...,p_m,u):[0,\infty)\times R^n\times...\times R^n\times U\to R^n$ satisfies the inequalities $\|F(s,p_1,...,p_m,u)\|\le\alpha(\|p_1\|+...+\|p_m\|)$, $\alpha=const.>0$;

$$\|F(s,p_1,...,p_m,u)-F(s,\overline{p}_1,...,\overline{p}_m,u)\|\le\beta(\|p_1-\overline{p}_1\|+...+\|p_m-\overline{p}_m\|),\ \beta=const.>0$$

2) $\|F(s,p_1,...,p_m,u)-F(s,p_1,...,p_m,\overline{u})\|\le\gamma\|u-\overline{u}\|$, $\gamma=const.>0$;

3) The initial function $\varphi(t,u):(-\infty,0]\times U\to R^n$ is continuous and $\varphi(0,u)=0$;

The delay functions $\Delta_k(t):[0,\infty)\to(-\infty,\infty)$ are continuous ones and satisfy the inequality $0<\Delta_0\le\Delta_k(t)\le t$.

4) $l(1+e^{-\mu\Delta_0})/\mu<1$; 5) $(l_u/\mu)<1$.

First we prove the following

Theorem 1. If conditions (BU) be fulfilled then the initial value problem (1), (2) has a unique solution $x\in M$ for every $u\in U$ and this solution is continuously depending on $u\in U$.

***Proof*:** Let us present an initial value problem

$$\dot{x}(t)=F(t,x(t-\Delta_1(t)),...,x(t-\Delta_m(t)),u),\quad t\in[0,\infty), \tag{4}$$

$$x(t)=\varphi_0(t),\quad t\in(-\infty,0].$$

in an equivalent operator form:

$$x(t)=\int_0^t F(s,x(s-\Delta_1(s)),...,x(s-\Delta_m(s)),u)ds,\ \ t\in[0,\infty) \tag{5}$$

$$x(t)=\varphi_0(t),t\le 0$$

Define an operator by the formula

$$B(x,u)(t)=\begin{cases}\int_0^t F(s,x(s-\Delta_1(s)),...,x(s-\Delta_m(s))u)ds, t\in[0,\infty)\\ \varphi_0(t),\quad t\in(-\infty,0]\end{cases}.$$

where the function $\varphi_0(t)$ is a continuous one. Its fixed point is a solution of (4).

In what follows we prove that B has a unique fixed point $x(u)$ that depends on u and $x(u)$ is a continuous map $x(u):U\to R^n$.

Let us introduce the set

$M=\{x(.)\in C(-\infty,\infty):x(.)\to R^n$ and $\|x(t)\|\le X_0e^{\mu t}$ and $x(t)=\varphi(t),t\in(-\infty,0]\}$,

where X_0 and μ are positive constants.

Introduce in M a family of pseudo-metrics

$\rho_K(x,\bar{x})=\sup\{\|x(t)-\bar{x}(t)\|e^{-\mu(t-a)}:t\in K=[a,b]\}$,

where K runs over all compact intervals $A=\{K=[a,b]\}$. It is easy to verify that M turns into a complete uniform space with saturated family of pseudometrics $\{\rho_K(x,\bar{x})\}_{K\in A}$ (Angelov, 2009).

In what follows we show that the operator B maps M into itself. Indeed, first we notice that $B(x,u)(t)$ is a continuous function in t. Further on, for sufficiently large $\mu>0$ and $t\in(-\infty,\infty)$ we have

$$\|B(x,u)(t)\|\le\int_0^t\|F(x(s-\Delta_1(s)),...,x(s-\Delta_m(s)),u)\|ds\le$$

$$\le\int_0^t\alpha(\|x(s-\Delta_1(s)\|+...+\|x(s-\Delta_m(s)\|)ds\le\alpha\int_0^t(X_0e^{\mu(s-\Delta_1(s))}+...+X_0e^{\mu(s-\Delta_m(s))})ds\le$$

$$\le\alpha X_0\int_0^t(e^{\mu s}e^{-\mu\Delta_0}+...+e^{\mu s}e^{-\mu\Delta_0})ds\le\alpha X_0me^{-\mu\Delta_0}\int_0^te^{\mu s}ds\le$$

$$\le\alpha X_0me^{-\mu\Delta_0}\frac{e^{\mu t}-1}{\mu}\le e^{\mu t}\frac{\alpha me^{-\mu\Delta_0}}{\mu}X_0\le X_0e^{\mu t}.$$

Consequently the operator B maps the set M into itself.

It remains to show that B is a contractive operator.

First we introduce the map $j:A\to A$ in the following way

$j:K=[a,b]\to\Delta(K)=convexclosure\,\Delta_1(K)\cup...\cup\Delta_m(K)=[a_{\Delta^1},b_{\Delta^1}]$,where

$\Delta_k(K)=\{t-\Delta_k(t):t\in K\}$. It is easy to see that $b_{\Delta_1}\le b$. Therefore if

$j^n(k)=j(j^{n-1}(k))=[a_{\Delta_n},b_{\Delta_n}]$ then $b_{\Delta_n}\le b$. We do not consider the intersection

of $[a_{\Delta_n},b_{\Delta_n}]$ with $(-\infty,0]$ because $\rho_{K_n}(x,\bar{x})=\rho_{K_n}(\varphi,\varphi)=0$ for

$t\in K_n=(-\infty,0]\cap[a_{\Delta^n},b_{\Delta^n}]$, that is, $\rho_{j^n(K)}(x,\bar{x})\le Q(x,\bar{x})<\infty$. The last inequality implies that the uniform space is j-bounded (Angelov, 2009). That is why, we consider the intervals $K\subseteq[0,\infty)$.

Indeed for $t\in K$ we have

$$\|B(x,u)(t)-B(\bar{x},u)(t)\|\le$$

$$\le\int_0^t\|F(x(s-\Delta_1(s))),...,x(s-\Delta_m(s)),u)-F(\bar{x}(s-\Delta_1(s)),...,\bar{x}(s-\Delta_m(s)),u)\|ds\le$$

$$\le l\int_0^t(\|x(s-\Delta_1(s)-\bar{x}(s-\Delta_1(s))\|+...\|x(s-\Delta_m(s)-\bar{x}(s-\Delta_m(s))\|)ds\le$$

$$\le l\int_0^t(\rho_{\Delta_1(K)}(x,\bar{x})e^{\mu(s-\Delta_1(s))}+...+\rho_{\Delta_m(K)}(x,\bar{x})e^{\mu(s-\Delta_m(s))})ds\le$$

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$$\leq lme^{-\mu\Delta_0}\int_0^t e^{\mu s}ds\ \rho_{\Delta(K)}(x,\bar{x})\ \leq e^{\mu t}\frac{lme^{-\mu\Delta_0}}{\mu}\rho_{j(K)}(x,\bar{x}).$$

It follows

$$\rho_K(B(x,u),B(\bar{x},u))\leq\frac{lme^{-\mu\Delta_0}}{\mu}\rho_{j(K)}(x,\bar{x}).$$

Consequently we have proved that the operator is contractive in the sense of (Angelov, 2009) for sufficiently large $\mu>0$.

Therefore there exists a unique fixed point of *B* that is a solution of (1). The result is valid for every fixed $u\in U$, that is, $x=x(u)$.

We show a continuity of the solution $x(u):R^n\to R^n$. Indeed, let us consider two solutions $x(u),\ x(\bar{u})$ of (5). Then

$$\|x(u)(t)-x(\bar{u})(t)\|\leq$$

$$\leq\int_0^t\|F(s,x(s-\Delta_1(s),u),...,x(s-\Delta_m(s),u),u)-F(s,x(s-\Delta_1(s),\bar{u}),...,x(s-\Delta_m(s),u),\bar{u})\|ds\leq$$

$$\leq\gamma\int_0^t\|u(s)-\bar{u}(s)\|ds\leq\gamma\int_0^t e^{\mu s}ds\rho(u,\bar{u})\leq\gamma\frac{e^{\mu t}-1}{\mu}\rho(u,\bar{u})\leq\gamma\frac{e^{\mu t}}{\mu}\rho(u,\bar{u}).$$

Therefore

$\rho_K(x(u),x(\bar{u}))\leq\frac{\gamma}{\mu}\rho_K(u,\bar{u})$ which implies the continuity of the function $x(u)$.

Theorem 1 is thus proved.

3. APPLICATION TO CONTROL THEORY

We apply the result obtained to the following control problem: to find such a solution $x(t)$

$$\dot{x}(t)=G(t,x(t-\Delta_1(t)),...,x(t-\Delta(t)),u)+\gamma u,\quad t\in(0,\infty),\qquad(7)$$
$$x(t)=\varphi(t),\quad t\in(-\infty,0]\,;\ \varphi(0)=0,$$
$$x(T)=x_T\ .$$

Since the initial value problem (7) is solvable for every $u\in U\subset R^n$ we have

$$x(t)=\int_0^t G(s,x(s-\Delta_1(s),u),...,x(s-\Delta_m(s),u),u)ds+\gamma\,t\,u\,.$$
$$x(t)=\varphi(t),\quad t\in(-\infty,0]\,;\ \varphi(0)=0,$$
$$x(T)=x_T\ .$$

We have to find such value $u\in U=\{u\in R^n:\|u\|_n\leq U_0\}$ that

$$x_T=\int_0^T G(s,x(s-\Delta_1(s),u),...,x(s-\Delta_m(s),u),u)ds+\gamma\,T\,u\,.$$

We solve the above equation with respect to *u*.

$$u = \frac{x_T}{\gamma T} - \frac{1}{\gamma T}\int_0^T G(s, x(s-\Delta_1(s),u),\dots, x(s-\Delta_m(s),u),u)ds\,.$$

Define the operator

$$A(u) := \frac{x_T}{\gamma T} - \frac{1}{\gamma T}\int_0^T G(s, x(s-\Delta_1(s),u),\dots, x(s-\Delta_m(s),u),u)ds : U \to U\,.$$

Let us assume that the following conditions (GU) be fulfilled:

1) $\|G(s, p_1,\dots, p_m, u)\| \le g\|u\|$;

2) $\dfrac{\|x_T\|}{\gamma T} + \dfrac{gU_0}{\gamma}\dfrac{e^{\mu T}-1}{\mu T} \le U_0$;

3) $\|G(s, p_1,\dots, p_m, u) - G(s, \bar{p}_1,\dots, \bar{p}_m, \bar{u})\| \le g_1\|p_1 - \bar{p}_1\| + \dots + g_m\|p_m - \bar{p}_m\| + g_u\|u - \bar{u}\|$

4) $\left(\dfrac{g_1 + \dots + g_m}{\mu} + \dfrac{g_u}{\gamma}\right)\dfrac{e^{\mu T}-1}{\mu T} < 1.$

where $g, g_u, U_0, \gamma, g_1,\dots, g_m, l_u, \mu$ be positive constants.

Theorem 2. Let conditions (GU) be fulfilled. Then the operator *A* has a unique fixed point $u_0 \in U$ such $x(t,u_0)$ is a solution of the boundary value problem (7).

Proof: We consider the set
$U = \{u \in R^n : \|u\| \le U_0\}$.

We show that the operator *A* maps U into itself. Indeed,

$$\|A(u)\| \le \frac{\|x_T\|}{\gamma T} + \frac{1}{\gamma T}\int_0^T \|G(s, x(s-\Delta_1(s),u),\dots, x(s-\Delta_m(s),u),u)\|ds \le$$

$$\le \frac{\|x_T\|}{\gamma T} + \frac{g}{\gamma T}\int_0^T \|u\|e^{\mu s}ds \le \frac{\|x_T\|}{\gamma T} + \frac{g\|u\|}{\gamma T}\frac{e^{\mu T}-1}{\mu} \le \frac{\|x_T\|}{\gamma T} + \frac{gU_0}{\gamma}\frac{e^{\mu T}-1}{\mu T} \le U_0$$

To obtain that A is a contractive operator we notice

$$\|A(u) - A(\bar{u})\| \le$$

$$\le \frac{1}{\gamma T}\int_0^T \|G(s, x(s-\Delta_1(s),u),\dots, x(s-\Delta_m(s),u),u) - G(s, x(s-\Delta_1(s),\bar{u}),\dots, x(s-\Delta_m(s),\bar{u}),\bar{u})\|ds \le$$

$$\le \frac{1}{\gamma T}\int_0^T \left(g_1\|x(s-\Delta_1(s),u) - x(s-\Delta_1(s),\bar{u})\| + \dots + g_m\|x(s-\Delta_m(s),u) - x(s-\Delta_m(s),\bar{u})\| + g_u\|u-\bar{u}\|\right)ds \le$$

$$\leq \frac{1}{\gamma T}\int_0^T \left(g_1 e^{\mu s}\rho_{\Delta(K)}(x(u),x(\bar{u})) + \ldots + g_m e^{\mu s}\rho_{\Delta(K)}(x(u),x(\bar{u}))_n + g_u e^{\mu s}\rho(u,\bar{u})\right)ds \leq$$

$$\leq \frac{g_1\rho_{\Delta(K)}(x(u),x(\bar{u})) + \ldots + g_m\rho_{\Delta(K)}(x(u),x(\bar{u}))_n + g_u\rho(u,\bar{u})}{\gamma}\frac{e^{\mu T}-1}{\mu T} \leq$$

$$\leq \frac{\left(g_1\frac{\gamma}{\mu}\rho_{\Delta(K)}(u,\bar{u}) + \ldots + g_m\frac{\gamma}{\mu}\rho_{\Delta(K)}(u,\bar{u})\right) + \frac{g_u}{\gamma}\rho_K(u,\bar{u})}{\gamma}\frac{e^{\mu T}-1}{\mu T} \leq$$

$$\leq \left(\frac{g_1}{\mu} + \ldots + \frac{g_m}{\mu} + \frac{g_u}{\gamma}\right)\frac{e^{\mu T}-1}{\mu T}\rho_K(u,\bar{u}).$$

We have used $\rho_{\Delta(K)}(u,\bar{u}) \leq \rho_K(u,\bar{u})$.

Consequently

$$\rho_K\left(A(u), A(\bar{u})\right) \leq \left(\frac{g_1 + \ldots + g_m}{\mu} + \frac{g_u}{\gamma}\right)\frac{e^{\mu T}-1}{\mu T}\rho_K(u,\bar{u}).$$

Then there exists a unique fixed point u_0 of *A*, that is, $u_0 = A(u_0)$.

Theorem 2 is thus proved.

4. DISCUSSION AND CONCLUSION

One can obtain an explicit approximated solution by successive approximations from the operator equation. Let us begin with some $u = u^{(0)} \in U$ and $x^{(0)} = x(t,u^{(0)})$ be a corresponding solution of (7). After substitution in $u = \frac{x_T}{\gamma T} - \frac{1}{\gamma T}\int_0^T G(s, x(s,u), x(s-\Delta(s),u), u)ds$ we obtain the sequences

$$x^{(1)}(t) = \int_0^t G(s, x^{(0)}(s-\Delta_1(s),u^{(0)}),\ldots, x^{(0)}(s-\Delta_m(s),u^{(0)}),u^{(0)})ds + \gamma t u^{(0)}$$

$$u^{(1)} = \frac{x_T}{\gamma T} - \frac{1}{\gamma T}\int_0^T G(s, x^{(1)}(s-\Delta_1(s),u^{(0)}),\ldots, x^{(1)}(s-\Delta_m(s),u^{(0)}),u^{(0)})ds,$$

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$$x^{(n+1)}(t) = \int_0^t G(s, x^{(n)}(s-\Delta_1(s),u^{(n)}),\ldots, x^{(n)}(s-\Delta_m(s),u^{(n)}),u^{(n)})ds + \gamma t u^{(n)}$$

$$u^{(n+1)} = \frac{x_T}{\gamma T} - \frac{1}{\gamma T}\int_0^T G(s, x^{(n+1)}(s-\Delta_1(s),u^{(n)}),\ldots, x^{(n+1)}(s-\Delta_m(s),u^{(n)}),u^{(n)})ds,$$

……………………………………………………………….. .

Denote by $\tilde{u} = \lim_{n\to\infty} u^{(n)}$ and $\tilde{x}(t,u) = \lim_{n\to\infty} x^{(n)}(t,u^{(n)})$ and then we substitute the obtained functions in the operator equation. So we obtain

$$\tilde{u} = \frac{x_T}{\gamma T} - \frac{1}{\gamma T}\int_0^T G(s, \tilde{x}(s-\Delta_1(s),\tilde{u}),\ldots, \tilde{x}(s-\Delta_m(s),\tilde{u}),\tilde{u})ds.$$

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REFERENCES

[1]. Angelov, V.G. 2014. A Method for Analysis of Transmission Lines Terminated by Nonlinear Loads. Nova Science. New York.
[2]. Angelov, V.G. 1987. Fixed point theorem in uniform spaces and applications. Czechoslovak Math. J., vol. 37 (112), 19-33.
[3]. Angelov, V.G. 1981. Some neutral equations with a control parameter. Bull. Austral. Math. Soc., 23, №3, 383-394.
[4]. Angelov, V.G. 2009. Fixed Points in Uniform Spaces and Applications. Cluj University Press, Cluj-Napoca, Romania.
[5]. Cipriano, A. Industrial Products for Advanced Control of Mineral Processing Plants. Advanced Control and Supervision of Mineral Processing Plants. Editors D. Sbarbaro.
[6]. Mossiagin, V.V. 1970. Monotone operator method for two-point boundary value problem for differential equations in Banach spaces. Proceedings of Petrozavodsk University, Mathematical Sciences, v.18, No.2, 128-134, (in Russian).
[7]. Parashkevova, D.D. 2006. Analysis of Technological Solutions and Methods for Control of Copper Ores Flotation. PhD Thesis, University of Mining and Geology „St. I. Rilski", Sofia, (in Bulgarian).
[8]. Prozuto, V.S. 1987. Automation System for Control of Technological Processes in Flotation Plants. Moscow, (in Russian).
[9]. Speedy, C.R., Brown, R.F., Goodwin, G.C. 1970. Control Theory. Oliver and Boyd, Edinburgh.
[10]. Wills, B.A., Finch, J.A. 2016. Wills' Mineral Processing Technology. Eighth Edition, Elsevier.